2018 IMO P4

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Solution to 2018 IMO P4

Problem: A *site* is any point (x, y) in the plane for which $x, y \in \{1, \ldots, 20\}$. Initially, each of the 400 sites is unoccupied. Amy and Ben take turns posting stones on unoccupied site, with Amy going first; Amy has the additional restriction that no two of her stones may be at distance equal to $\sqrt{5}$. They stop once either player cannot move. Find the greatest K such that Amy can ensure that she places at least K stones.

Solution: The answer is $K = \boxed{100}$.

Proof that Amy can place at least $K = 100$ *stones.* Color the board like a chessboard. Amy can place stones only on black squares (she doesn't have to worry about the $\sqrt{5}$ restriction). She can make 100 moves before she runs out of black squares.

Proof that Ben can stop Amy from placing more than 100 *stones.* Break the region into 4×4 squares, and color them as follows. The number n denotes the nth color we have.

1 2 **3** 4

3 4 1 2

2 1 4 3

4 **3** 2 1

Call a group of stones with the same color within a 4×4 square a *loop*.

Any time Amy places a stone, Ben can place a stone *at the opposite place in the same loop*. An example is in bold.

This blocks off all the stones in the loop that Amy just placed a stone in. Therefore, Amy can place a maximum of one stone in each loop. The maximum number of stones she can place is the number of loops, which is 100.

This completes the problem.