

# 2011 JMO P2

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## Solution to 2011 JMO P2

**Problem:** Let  $a, b, c$  be positive real numbers such that  $a^2 + b^2 + c^2 + (a + b + c)^2 \leq 4$ . Prove that

$$\frac{ab + 1}{(a + b)^2} + \frac{bc + 1}{(b + c)^2} + \frac{ca + 1}{(c + a)^2} \geq 3.$$

**Solution:** We know that  $a^2 + b^2 + c^2 + ab + bc + ca \leq 2$ . Note that

$$\begin{aligned} & \sum_{cyc} \frac{2ab + 2}{(a + b)^2} \\ & \geq \sum_{cyc} \frac{2ab + (a^2 + b^2 + c^2 + ab + bc + ca)}{(a + b)^2} \\ & = \sum_{cyc} \frac{(a + b)^2 + (c + a)(c + b)}{(a + b)^2} \\ & = 3 + \sum_{cyc} \frac{(c + a)(c + b)}{(a + b)^2}. \end{aligned}$$

By AM-GM, this is

$$\geq 3 + 3\sqrt[3]{1} = 6.$$

This completes the problem.