

2000 IMO P2

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Solution to 2000 IMO P2

Problem: Let a, b, c be positive real numbers with $abc = 1$. Show that

$$\left(a - 1 + \frac{1}{b}\right)\left(b - 1 + \frac{1}{c}\right)\left(c - 1 + \frac{1}{a}\right) \leq 1.$$

Solution: Let

$$a = \frac{x}{y}$$

$$b = \frac{y}{z}$$

$$c = \frac{z}{x}.$$

We want to show that

$$\left(\frac{x + z - y}{y}\right)\left(\frac{y + x - z}{z}\right)\left(\frac{z + y - x}{x}\right) \leq 1,$$

or

$$(-x + y + z)(x - y + z)(x + y - z) \leq xyz.$$

Expanding gives Schur's inequality.